Al Moattasem International School

Jubail

<u>Level 9 – Revision Work Sheet - 5</u>

Chapter 8

<u>Topic - Composite Functions & Inverse Functions</u>

<u>Answer Key</u>

1) <u>Solution</u>

First draw a flow diagram for the function.

$$\xrightarrow{x} \boxed{\text{multiply by } 4} \xrightarrow{4x} \boxed{\text{subtract } 3} \xrightarrow{4x-3} \boxed{\text{divide by } 2} \xrightarrow{\frac{4x-3}{2}}$$

Now draw a flow diagram, **starting from the right**, with each operation replaced by its inverse.

$$\xleftarrow{\frac{2x+3}{4}} \boxed{\text{divide by } 4} \xleftarrow{2x+3} \boxed{\text{add } 3} \xleftarrow{2x} \boxed{\text{multiply by } 2} \xleftarrow{x}$$

The inverse of $h : x \mapsto \dfrac{4x-3}{2}$ is thus $h^{-1} : x \mapsto \dfrac{2x+3}{4}$.

2 a) Solution

For the function $f : x \mapsto 3x + 4$ the flow diagram is

$$\xrightarrow{x} \boxed{\text{multiply by } 3} \xrightarrow{3x} \boxed{\text{add } 4} \xrightarrow{3x+4}$$

The inverse is thus

$$\xleftarrow{\frac{x-4}{3}} \boxed{\text{divide by } 3} \xleftarrow{x-4} \boxed{\text{subtract } 4} \xleftarrow{x}$$

so

$$f^{-1} : x \mapsto \frac{x-4}{3}\,.$$

2b) Solution

For the function $f : x \mapsto 4(x - 1)$ the flow diagram is

$$\xrightarrow{x} \boxed{\text{subtract } 1} \xrightarrow{x-1} \boxed{\text{multiply by } 4} \xrightarrow{4(x-1)}$$

The inverse is thus

$$\xleftarrow{\frac{x}{4}+1} \boxed{\text{add } 1} \xleftarrow{\frac{x}{4}} \boxed{\text{divide by } 4} \xleftarrow{x}$$

Thus

$$f^{-1} : x \mapsto \frac{x}{4} + 1\,.$$

2c) Solution

For the function $f : x \mapsto 3(2x+5)$ the flow diagram is

$$\xrightarrow{x} \boxed{\text{multiply by } 2} \xrightarrow{2x} \boxed{\text{add } 5} \xrightarrow{2x+5} \boxed{\text{multiply by } 3} \xrightarrow{3(2x+5)}$$

The inverse has the flow diagram

$$\xleftarrow{\frac{1}{2}\left(\frac{x}{3}-5\right)} \boxed{\text{divide by } 2} \xleftarrow{\frac{x}{3}-5} \boxed{\text{subtract } 5} \xleftarrow{\frac{x}{3}} \boxed{\text{divide by } 3} \xleftarrow{x}$$

Thus

$$f^{-1} : x \mapsto \frac{1}{2}\left(\frac{x}{3}-5\right)$$

2d) Solution

For the functions $g : x \mapsto -8x+3$ the flow diagram is

$$\xrightarrow{x} \boxed{\text{multiply by } -8} \xrightarrow{-8x} \boxed{\text{add } 3} \xrightarrow{-8x+3}$$

The inverse function has flow diagram

$$\xrightarrow{-\frac{1}{8}(x-3)} \boxed{\text{divide by } -8} \xrightarrow{x-3} \boxed{\text{subtract } 3} \xrightarrow{x}$$

so the inverse function is

$$g^{-1} : x \mapsto -\frac{1}{8}(x-3)\,.$$

2e) Solution

Before beginning this question the function may be simplified by noting that

$$\frac{1}{2}(3x+4)+6=\frac{1}{2}(3x+4)+\frac{12}{2}=\frac{1}{2}(3x+16)$$

so that the function is $g : x \mapsto \frac{1}{2}(3x+16)$. The flow diagram for this function is

$$\xrightarrow{x} \boxed{\text{multiply by } 3} \xrightarrow{3x} \boxed{\text{add } 16} \xrightarrow{3x+16} \boxed{\text{divide by } 2} \xrightarrow{\frac{1}{2}(3x+16)}$$

The inverse flow diagram is

$$\xleftarrow{\frac{1}{3}(2x-16)} \boxed{\text{divide by } 3} \xleftarrow{2x-16} \boxed{\text{subtract } 16} \xleftarrow{2x} \boxed{\text{multiply by } 2} \xleftarrow{x}$$

and $$g^{-1} : x \mapsto \frac{1}{3}(2x-16).$$

2f) Solution

For the functions $g : x \mapsto \dfrac{2x+1}{3}$ the flow diagram is

$$\xrightarrow{x} \boxed{\text{multiply by } 2} \xrightarrow{2x} \boxed{\text{add } 1} \xrightarrow{2x+1} \boxed{\text{divide by } 3} \xrightarrow{\frac{1}{3}(2x+1)}$$

and the inverse is given by

$$\xleftarrow{\frac{1}{2}(3x-1)} \boxed{\text{divide by } 2} \xleftarrow{3x-1} \boxed{\text{subtract } 1} \xleftarrow{3x} \boxed{\text{multiply by } 3} \xleftarrow{x}$$

so that

$$g^{-1} : x \mapsto \frac{1}{2}(3x-1).$$

3a) Solution

For the functions $f:(x) = 2x+1$, $g(x) = x-3$ the function fg is

$$fg : x \mapsto 2(x-3)+1 = 2x-6+1 = 2x-5\,.$$

The flow diagram is

The function fg can also be determined as follows. The two functions can be written as $f(z) = 2z+1$ and $g(x) = x-3$. Then by substituting $z = g(x)$ into $f(z) = 2z+1$,

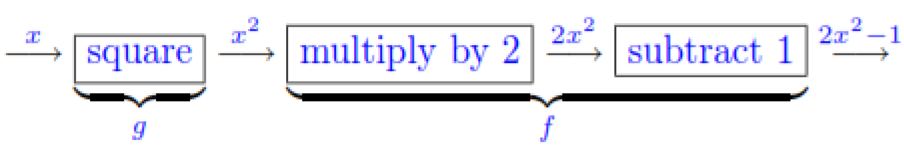
$$\begin{aligned} fg(x) &= f(g(x)) \\ &= 2g(x)+1 \\ &= 2(x-3)+1 \\ &= 2x-5\,. \end{aligned}$$

3b) Solution

For the functions $f : x \mapsto 2x-1$, $g : x \mapsto x^2$ the function fg is

$$fg : x \mapsto 2x^2 - 1\,.$$

The flow diagram is



The composition may also be determined by writing $f(z) = 2z-1$ and $g(x) = x^2$ and substituting $z = g(x)$, obtaining

$$\begin{aligned} fg(x) &= f(g(x)) \\ &= 2g(x)+1 \\ &= 2\left(x^2\right)-1 \\ &= 2x^2-1\,. \end{aligned}$$

3c) Solution

For the functions $f : x \mapsto x^2$, $g : x \mapsto 2x - 1$ the function fg is

$$fg : x \mapsto (2x-1)^2\,.$$

The flow diagram is

$$\xrightarrow{x} \underbrace{\boxed{\text{multiply by } 2} \xrightarrow{2x} \boxed{\text{subtract } 1}}_{g} \xrightarrow{2x-1} \underbrace{\boxed{\text{square}}}_{f} \xrightarrow{(2x-1)^2}$$

Alternatively, writing $f(z) = z^2$ and $g(x) = 2x - 1$, then substituting for $z = g(x)$:

$$\begin{aligned} fg(x) &= f(g(x)) \\ &= (g(x))^2 \\ &= (2x-1)^2\,. \end{aligned}$$