CHAPTER – 8

SETS , VECTORS AND FUNCTIONS

- **Topic – Vectors**

- **Vectors in Geometry**

In geometry problems involving vectors, the vectors can be written using a pair of capital letters with an arrow above them.

Vectors

A vector quantity has both size and direction. Vectors can be added, subtracted and multiplied by a scalar. Geometrical problems can be solved using vectors.

Vector addition and subtraction

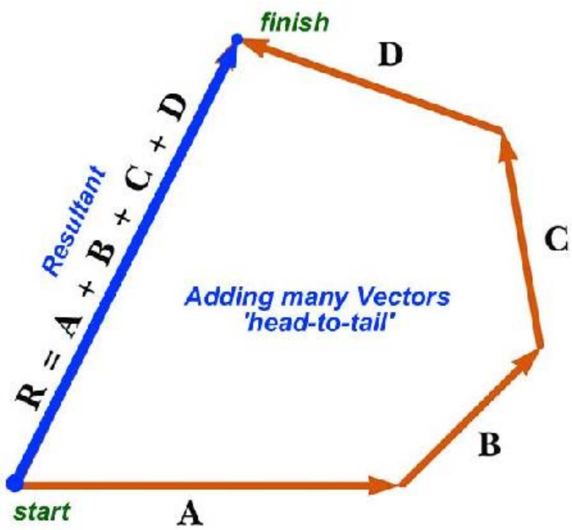
When 2 vectors are added or subtracted the vector produced is called the resultant.

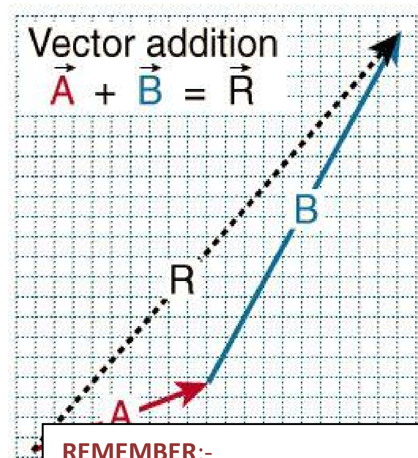
The resultant is identified by a double arrowhead.

To obtain the resultant vector a + b, the tail of b is joined to the nose of a.

To obtain the resultant vector b + a, the tail of a is joined to the nose of b.

So adding “nose to tail” or “tail to nose” gives the same resultant vector.





REMEMBER:-

Go via a route of known vectors

Multiplication by a Scalar

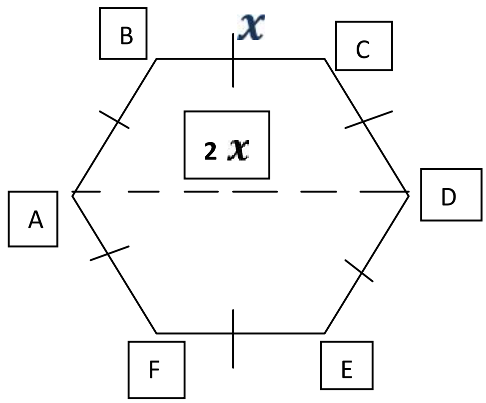
- **Ordinary numbers are scalars**
- **Scalars are easy to use. Just treat them as normal numbers.**
- **Scalars have magnitude but no direction. Vectors can be multiplied by a scalar to produce another vector**

When **x** is multiplied by −3 the result is −3**x**.

Note:
(1) The negative sign reverses the direction of the vector.

<u>Relationship between hexagon and vector</u>

Remember: A hexagon is a six-sided polygon or 6-gon. The word hexagon comes from the Greek 'hex', meaning six, and 'gonia', meaning corner or angle.

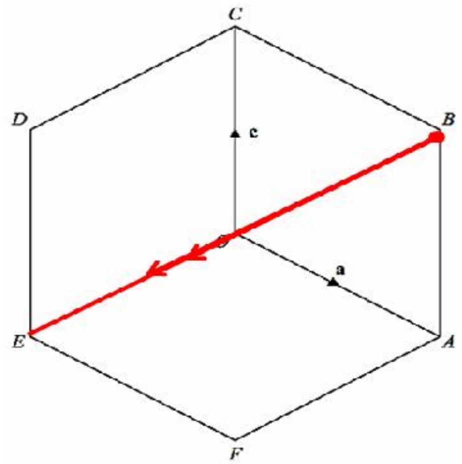


1) Opposite sides of a regular hexagon are parallel
   Side AB parallel to side DE
   Side BC parallel to side EF
   Side CD parallel to side FA

2) For a regular hexagon all sides are equal

3) **For a regular hexagon. Given side BC = x then diagonal AD = 2x**

4)



In the figure, we should take note of the following vectors:

1) $\overrightarrow{BE} = 2\,\overrightarrow{BO} = 2\overrightarrow{OE}$
2) $\overrightarrow{OA} = \overrightarrow{CB} = \mathbf{a}$
3) $\overrightarrow{OC} = \overrightarrow{AB} = \mathbf{c}$

A vector between two points A and B is described as: $\overrightarrow{AB}$, or $\mathbf{a}$

The vector can also be represented by the **column vector** $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

The top number is how many to move in the positive x -direction and

the bottom number is how many to move in the positive y -direction.

Vectors are equal if they have the same magnitude and direction regardless of where they are. Example:-

$\overrightarrow{CD} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \overrightarrow{EF} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

So $\overrightarrow{CD} = \overrightarrow{EF}$

Vector arithmetic

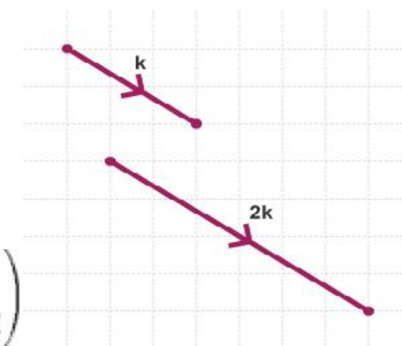
Multiplying vectors by a scalar

Vectors can be multiplied by a scalar which changes the size of the vector but not the direction.

$$k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

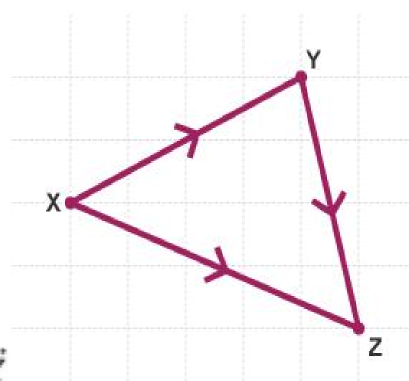
The vector $2k$ is twice as long as the vector k. Double each number in k to get $2k$.

$$2k = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$



Adding vectors

Vectors can be added by drawing the first vector, then starting the second vector where the first vector ends.



$$\overrightarrow{XY} + \overrightarrow{YZ} = \overrightarrow{XZ}$$

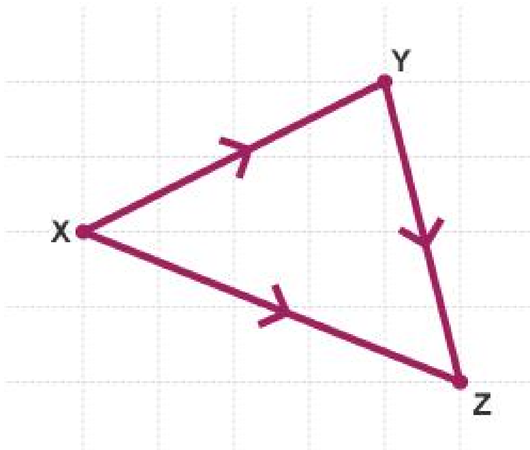
$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 4+1 \\ 2+-4 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

The single vector they create ($\overrightarrow{XZ}$) is the resultant vector.

Travelling from X to Y, then from Y to Z, is the same as travelling from X to Z.

Subtracting vectors

Subtracting a vector is the same as adding a negative vector.



$\overrightarrow{YX} + \overrightarrow{XZ} = \overrightarrow{YZ}$

Since the vector $\overrightarrow{YX}$ has the same magnitude but opposite direction to the vector $\overrightarrow{XY}$:

$\overrightarrow{YX} = \overrightarrow{-XY}$

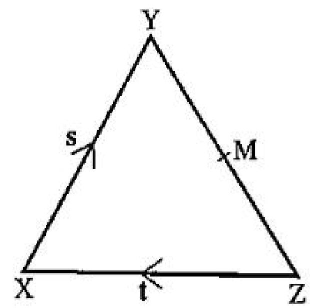
$\overrightarrow{-XY} + \overrightarrow{XZ} = \overrightarrow{YZ}$

$-\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -4+5 \\ -2+-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

<u>**Assignment**</u>   <u>**Exercise 6**</u>

7. In ΔXYZ, the mid-point of YZ is M.
If $\overrightarrow{XY} = \mathbf{s}$ and $\overrightarrow{ZX} = \mathbf{t}$, find $\overrightarrow{XM}$ in terms

<u>Solution</u>



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$\overrightarrow{XY} = s$

$\overrightarrow{ZX} = t$

YM =MZ (Since M is the midpoint of YZ)

To find :-

$\overrightarrow{XM} = \overrightarrow{XY} + \frac{1}{2}\overrightarrow{YZ}$

$\overrightarrow{XM} = s + \frac{1}{2}\overrightarrow{YZ}$

<u>**So, to find $\overrightarrow{YZ}$**</u>

$\overrightarrow{YZ} = \overrightarrow{YX} + \overrightarrow{XZ}$

$\overrightarrow{YZ} = -s + (-t)$

$\overrightarrow{YZ} = -s - t$

<u>Now To find **$\overrightarrow{XM}$**:-</u>

$\overrightarrow{XM} = s + \frac{1}{2}\, YZ$

$\overrightarrow{XM} = \mathbf{s + \frac{1}{2}(-s - t)}$

$= \mathbf{s - \frac{1}{2}\, s - \frac{1}{2}\, t}$

$= \frac{2\ \ -s-t}{2}$

$= \frac{s-t}{2}$

$\overrightarrow{\mathbf{XM}} = \mathbf{\frac{1}{2}\ s - \frac{1}{2}\ t}$